The paper presents mathematical model of external ballistics of projectile, fired from cannon. The drag force has a decisive influence on the dynamics of the projectile. Direct calculation of the functional dependence of the drag force on the set of parameters is quite controversial and has not been utilized so far. Therefore, the author calculates the functional relativity, approximately, solving the inverse problem of mechanics. The flight range of a projectile at certain aiming angles, noted in the firing tables, determine the drag force according to the effect on a projectile of its weight and Coriolis force. Its functional dependence is described separately at the stages of projectile’s motion with supersonic, transonic and subsonic velocities. Due to the functional dependence of the drag force of a projectile, the influence of a charge projectile, initial velocity, atmospheric pressure, air temperatures and projectile’s mass effect on kinematic parameters of its motion can be calculated. Knowledge of this dependence of the drag force of the projectile’s movement allows to automate, using appropriate software, the determination of the elevation angle depending on the location of the target and the values of deterministic and nondeterministic factors.

**Keywords:** cannon, drag force, dynamics of projectile motion, external ballistics.

### Introduction

The statement of the problem. One of the fundamental tasks while firing cannon and howitzer is to establish the relationship between the angle of elevation and the location of the target. It depends on the deterministic (shape and mass of a projectile, atmospheric pressure, air density and temperature, derivation), non-deterministic (muzzle speed, value and direction of wind speed) and other factors. The research at the training area has resulted in designing of firing tables for each type of armament and the corresponding projectile. They present a discrete relationship between angle of elevation and firing range under standard conditions. However, shooting is carried out at any distance, the target is not always located in the range of the weapon, deterministic and non-deterministic factors could vary. In this case, calculating of angle of elevation using firing tables at which the projectile reaches the target is a time-consuming procedure and the angle is determined with a certain approximation. Establishing an analytical relationship between the angle of elevation and the coordinates of the target location, for all values of deterministic and non-deterministic factors, has not yet been achieved.

Analysis of recent researches and publications. A significant amount of bibliographic information and the basics of theoretical research on the external ballistics of bullets and projectiles are presented, for instance, in [1–4]. The above-mentioned ones, including other scientific studies [5–11] define the drag force by the relation

\[ R = \frac{\rho V^2 \pi d^2}{2} C_D \cdot \]

where
\[ R \] – the drag force;
\[ V \] – projectile velocity;
\[ \rho \] – air density;
\[ V_s \] – speed of sound in the air;
\[ d \] – projectile caliber;
\[ i \] – projectile’s form-factor,
\[ c_x \left( \frac{V}{V_s} \right) \] – frontal drag function;
\[ C_D \] – drag force coefficient.

The frontal drag function for a certain type of projectile is determined by conducting experimental studies and it has a discrete character. The projectile’s form-factor \( i \) is calculated based on the comparison of the geometric parameters of this projectile with some reference one.

The authors’ research of the external ballistics of a 7,62 mm bullet fired from an AKM (modernized Kalashnikov automatic rifle) with initial velocity \( V_0 \) and mass \( m \) \((V_0 = 715 \text{ m/s}, m = 0,0079 \text{ kg})\) and RPK (Kalashnikov light machine gun) \((V_0 = 745 \text{ m/s}, m = 0,0079 \text{ kg})\) small arms allows us to assert that although the same bullet is used, the functional dependence of the drag forces on the speed of the bullet are different. A similar result is observed for 7,62 mm caliber bullets fired from SVD (Dragunov sniper rifle) \((V_0 = 830 \text{ m/s}, m = 0,0096 \text{ kg})\), PK (Kalashnikov machine gun) \((V_0 = 825 \text{ m/s}, m = 0,0096 \text{ kg})\) and PKT (Kalashnikov tank machine gun) \((V_0 = 855 \text{ m/s}, m = 0,0096 \text{ kg})\).
Therefore, the use of the projectile’s form-factor during calculations significantly worsens the description of the dynamics of the bullets or projectiles. We assume that the projectile’s form-factor can be used if the shot is fired from a smooth-bore weapon, and not from a rifled one. The frontal drag function $c_d \left( \frac{V}{V_a} \right)$ in formula (1) is preferably replaced by the drag force coefficient $C_D$. In this case the formula (2) is used. The drag force coefficient can be described by the dependence

$$C_D = C_{D_0} + C_{D_2} \delta^2,$$

where $C_{D_0} = \text{const}$, $C_{D_2} = \text{const}$ and $\delta$ – yaw angle.

**Aim of the paper.** The crucial influence on the projectile’s dynamics is triggered by the drag force. Based only on analytical methods, it is problematic to find its functional dependence on projectile’s velocity, deterministic or nondeterministic factors. The paper proposes a mathematical model of its calculus on a combination of theoretical and experimental research. Due to the functional dependence of the projectile’s drag force, using appropriate software, the value of the angle of elevation, which ensures the flight of the projectile to the target location at specific values of deterministic and non-deterministic factors, could be determined. Consequently, the accuracy of fire could be enhanced.

Thus, our aim is to develop a calculation method of the functional relationship of the projectile’s drag force, taking into consideration the results of the research conducted in the training area, which are provided in the firing tables, as well as to make comparative analysis of the outcomes obtained from theoretical research and practical experiments.

**Discussion**

According to experimental studies, it is stated that the magnitude of the drag force of a projectile is:

– proportional to its velocity of a certain degree, not necessarily the squared one;

– considerably depends on whether the velocity of a projectile is supersonic, transonic or subsonic.

Therefore, motion of the projectile in the air contains the following most likely combinations of stages:

– motion of the projectile at supersonic velocity;

– motion of the projectile at transonic velocity;

– motion of the projectile at subsonic velocity.

Hence, it is proposed to determine the functional dependencies of the value of the projectile’s drag force for each stage. Dependencies at each stage will be described by a formula:

$$R(t) = c_x \cdot \rho_a \cdot s_x \cdot (V(t))^{2+\beta_i} \left( \frac{V(t)}{V_a(t)} \right)^{\beta_i},$$

where $c_x$ – a coefficient that takes into account the aerodynamic shape of the projectile, with its longitudinal airflow, and proportionality;

$\rho_a$ – air density;

$s_x$ – the maximum cross sectional area of the projectile;

$V(t)$ – projectile velocity at any stage;

$V_a(t)$ – the magnitude of the velocity of sound in the air;

$\gamma_i$ (i = 1, 2, 3) and $\beta_i$ (i = 1, 2, 3) – the coefficients, the values of which are different at supersonic (i = 1), transonic (i = 2) and subsonic (i = 3) velocities.

The origin of the coordinate system $Oxyz$ is set up at the launch point of the projectile. The axis $Ox$ places in the horizontal plane of the weapon and is aimed in the target’s direction, the axis $Oz$ is directed vertically upwards, and the axis $Oy$ is directed perpendicular to the plane $Oxz$, forming a right-handed rectangular coordinates system $Oxyz$.

The drag force (3) depends on the air density $\rho_a$. Based on the ideal gas law, the equation for air takes the following form

$$pV_a = \frac{m_a R_{un} T}{\mu_a},$$

where $p$ – an atmospheric air pressure;

$V_a$ – its volume;

$m_a$ – the mass of the gas;

$R_{un} = 8314 \ J/(\text{kmol} \cdot \text{K})$ – universal gas constant;

$T$ – absolute air temperature;

$\mu_a = 28.96 \ \text{kg/kmol}$ – apparent molar mass of air.

Since the air density is determined by the equation

$$\rho_a = \frac{m_a}{V_a},$$

so, using dependence (4), we obtain

$$\rho_a(z) = \frac{\rho(T_z) \mu_a}{R_{un} T(z)},$$

where $z$ is projectile’s motion altitude above the horizontal plane of the weapon and its dimension $[z] = m$.

Pursuant to experimental studies, it is established that the air temperature decreases on average 0.6328 °C to a change in altitude of 100 meters up [1]. Thus, the alteration in absolute air temperature up to altitude change is described by the equation

$$T(z) = TK - 0.006328 z,$$

where $TK$ is the absolute temperature of the air at the location point of the weapon.

International barometric formula says how atmospheric pressure varies with a change in altitude. It has the following entry when the pressure is equal to
760 mm Hg and the temperature is equal to 15 °C:

\[ p_0(z) = 101325 \left(1 - \frac{6.5 z}{288000}\right)^{5.255}, \tag{7} \]

where \( z \) is an altitude above sea level and dimension \([z]\) = m.

Since the results of the experiments at the training area, which are recorded in the firing tables of, are given at a pressure of 750 mm Hg, the equation (7) will be the following one:

\[ p(z) = 101325 \left(1 - \frac{6.5(z + z_p)}{288000}\right)^{5.255}, \tag{8} \]

where \( z_p \) magnitude is equals

\[ z_p = \frac{288000}{6.5} \left[1 - \left(\frac{750-133,222}{101325}\right)^{5.255}\right] = 111.54 \text{ m}. \]

In view of dependencies (5), (6) and (8) it is determined that air density varies in accordance with the altitude \( z \) due to the law

\[ \rho_a(z) = \frac{101325 \rho_a}{R_{atm}(TK - 0.006328z)} \left(1 - \frac{6.5(z + z_p)}{288000}\right)^{5.255} \tag{9} \]

The velocity of sound in an ideal gas depends on the temperature of the gas and its value is determined by the formula [2]

\[ V_s = \sqrt{\frac{k_a R_{atm} T(z)}{\mu_a}}, \tag{10} \]

where \( k_a \) is an adiabatic index and for air \( k_a = 1.4 \).

Taking into account (6) and (10) we conclude that the change of the sound velocity in the air due to the altitude will be calculated by the following dependence

\[ V_s(z) = \sqrt{\frac{k_a R_{atm}}{\mu_a} (TK - 0.006328 z)} \tag{11} \]

Based on the dependencies (3), (9) and (11) we deduce that the functional dependence of the drag force to the motion of the projectile, depending on its altitude and velocity, will take the form

\[ R(t) = \frac{c_s s a \cdot 101325}{R_{atm} (TK - 0.006328 z)} \left(1 - \frac{6.5(z + z_p)}{288000}\right)^{5.255} \times \]

\[ \times \left[ \frac{(V(t))^{2 + \gamma_1 + \beta_1}}{(V(t))^{0.5 \beta_1}} \right]. \tag{12} \]

The projectile’s movement in the air under the effect of the projectile weight \( \vec{P} \), the air resistance force \( \vec{R} \) and Coriolis force \( \vec{F}_{cor} \) is considered. The equation of this motion will be composed on the Newton’s second law of dynamics as an equation

\[ m \ddot{a} = \vec{P} + \vec{R} + \vec{F}_{cor}, \tag{13} \]

where \( m \) – a mass;
\( \ddot{a} \) – an acceleration of the projectile;
\( \vec{P} = m \vec{g} \) and \( g = 9.8 \text{ m/s}^2 \);
\( \vec{F}_{cor} = -m \ddot{a}_{cor} \);
\( \ddot{a}_{cor} = 2 \omega_e \times \vec{V} \) – the Coriolis acceleration;
\( \omega_e \) – a vector of angular velocity of the Earth’s rotation.

By projecting equation (13) on the coordinate axis, we obtain a system of ordinary differential equations

\[ \begin{align*}
    m \ddot{x} &= -R \cos \theta - 2 \omega_e m (\dot{z} \cos \lambda \cdot \cos \psi - \dot{y} \sin \lambda) \quad (14) \\
    m \ddot{y} &= -2 \omega_e m (\dot{x} \sin \lambda - \dot{z} \cos \lambda \cdot \sin \psi) \quad (15) \\
    m \ddot{z} &= -P - R \sin \theta \end{align*} \tag{16} \]

where \( \theta \) – the angle of inclination of the velocity vector of the projectile to the horizon at any time;
\( \lambda \) – latitude of the Earth where the firing does take place;
\( \psi \) – the angle between the firing direction and the eastern direction (during all calculations we put \( \lambda = 50^\circ \) and \( \psi = 90^\circ \)).

Discrete dependencies between the angle of departure \( \theta_i \) and the coordinate of the level point of the projectile motion \( x(\theta_i) \), which were obtained during the field research, are given in the firing tables

\[ x(\theta_2) = 200, \ x(\theta_3) = 400, \ x(\theta_4) = 600, \ x(\theta_5) = 800, \ x(\theta_6) = 1000, \ldots \tag{17} \]

For instance, the dynamics of the projectile OF-540 fired from a 152-mm howitzer D-20, full charge is considered. Its initial speed is supersonic and equal to \( V_0 = 655 \text{ m/s} \).

The system of differential equations (14)–(16) must be solved due to the initial data:

\[ x(0) = 0, \ y(0) = 0, \ z(0) = 0, \ \dot{x}(0) = 0, \ \dot{y}(0) = 0, \ \dot{z}(0) = V_0 \sin \theta_i, \] \tag{18} \]

where \( V_0 \) is an initial projectile’s velocity. We denote by \( \theta_j \) angle of departure, the magnitude of which is determined as

\[ \theta_j = \alpha_i + \gamma_{ver}, \tag{19} \]

where \( \alpha_i \) – angle of elevation;
\( \gamma_{ver} \) – vertical jump angle.

The functional dependence of the value of the drag force to the projectile’s movement (3) at the stage of its motion at supersonic speed is uncertain, because the values of the coefficients \( c_s, \gamma_1, \beta_1 \) are unknown. To determine them, you need to solve the inverse problem of dynamics. That is, taking into account the system of differential equations (14)–(16), the initial data (18) and the discrete relationship between the angle of departure...
\( \theta_i \) and the coordinate of the level point of the projectile \( x(\theta_i) \) (17) to determine the values of the coefficients \( c_x, \gamma_1, \) and \( \beta_1 \). The magnitudes of values \( c_x, \gamma_1, \) and \( \beta_1 \) were calculated with the method of successive approximations. Firstly, arbitrary values of \( c_x \) and \( \gamma_1 \) are set and the value \( \beta_1 \) is selected so that their combination provides a slight discrepancy between the theoretical values of the coordinates of the level points with the data specified in the firing tables [12], so there is

\[
x(\theta_2) = 200, \ x(\theta_3) = 400, \ x(\theta_4) = 600, \ x(\theta_{39}) = 7600. \quad (20)
\]

Providing a significant discrepancy between the theoretical and tabular values of the horizontal range of the projectile, other values \( c_x \) and \( \gamma_1 \) were taken consequently the value \( \beta_1 \) was picked again. Having accomplished the first two steps, the tendency to change the values of \( c_x, \gamma_1, \) and \( \beta_1 \) becomes obvious. Proceeding on analogical calculations, the values \( c_x, \gamma_1, \) and \( \beta_1 \) are determined, in case there is a slight discrepancy between the theoretical and tabular values of the total horizontal range of the projectile.

The following figures were taken for calculations: projectile’s weight – \( m = 43.56 \) kg; initial velocity – \( V_0 = 655 \) m/s; cross section area – \( s_x = \pi \cdot 0.077^2 \) m²; air temperature – \( t = 15 \) °C; atmospheric pressure – \( p = 750 \) mm Hg; vertical jump angle – \( \gamma_{ver} = 0 \).

The values of the coefficients were determined by the method of successive approximations \( c_x = 0.35, \gamma_1 = -0.097 \) and \( \beta_1 = -0.371 \).

In table 1 and fig. 1 it is indicated the kinematic parameters of the projectile’s motion if it flies only at supersonic speed. Analogous projectile’s movement parameters are specified if during its flight the speed changes from supersonic to transonic (Table 2 and Fig. 2), and from supersonic to transonic and subsonic (Table 3 and Fig. 3).

| Table 1 |
|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|
| \( \alpha_i, (i = 1,39), \) | \( t_{ik}, \) | \( \theta_{ic}, \) | \( x_{ik}, \) | \( \dot{x}_{ik}, \) | \( \ddot{x}_{ik}, \) | \( V_{ik}, \) | \( H_i, \) |
| degree | s | degree | m | m/s | m/s | m/s | m |
| \( \alpha_1=0^\circ00' \) | 0,0 | 0'0' | 0,0 | 655,0 | 0,0 | 655,0 | 0,0 | 0,0 | 0,0 |
| \( \alpha_2=0^\circ42' \) | 1,6101 | 0'44' | 1014,11 | 605,72 | -7,80 | 605,77 | 3,18 | 513,7 | 1014 |
| \( \alpha_3=1^\circ57' \) | 4,3853 | 2'14' | 2590,30 | 533,10 | -20,81 | 533,51 | 23,59 | 1339 | 2591 |
| \( \alpha_4=7^\circ57' \) | 16,3083 | 12'11' | 7602,59 | 335,70 | -72,48 | 343,44 | 327,2 | 4213 | 7641 |

Source: developed by the authors.

Fig. 1. Varying in projectile velocity and drag force at the angle of elevation \( \alpha_39 = 7^\circ57' \):
a) – varying in projectile velocity; b) – varying in projectile drag force

Source: developed by the authors based on formulas (14)–(16) for a) and formula (12) for b.
Note that these tables show the results obtained by the method proposed by the authors and in parentheses from the firing tables [12].

In table 1 there are designations:

- \( \alpha_i \) – angle of elevation;
- \( t_{ik} \) – duration of flight of the projectile;
- \( x_{ik} \) – horizontal coordinate of the level point of the projectile trajectory;
- \( \theta_{ic} \) – angle of fall;
- \( \dot{x}_{ik} \) – projection of velocity on the axis Ox;
- \( \dot{z}_{ik} \) – projection of velocity on the Oz axis;
- \( V_{ik} \) – final velocity of the projectile;
- \( H_i \) – apex of the trajectory;
- \( x_{iH} \) – horizontal distance to the apex of the trajectory;
- \( S_i \) – the length of the trajectory of the projectile.

The values of the projectile parameters from the firing tables [12] are given in parentheses.

Note that in this and the following tables, only the corresponding values for the beginning and end of the stage, the largest positive and negative deviations between the theoretical and tabular values of the level point of the projectile trajectory are indicated.

In fig. 1 \( V_s \) is a speed of sound under normal conditions; \( t \rightarrow 0 \) – zeroing of the projectile’s trajectory (duration of its flight).

On these and all subsequent graphs the dimension of time is \( [t] = s \), velocity is \([V] = m/s\) and the drag force is \([R] = N\).

Values of coefficients \( c_x = 0.35 \), \( \gamma_1 = -0.097 \) and \( \beta_1 = -0.371 \) provided a slight discrepancy between the theoretical and experimental results of the full horizontal flight range of the projectile at the stage of its motion at supersonic speed. The presence of discrepancies is possible because the experimental values were obtained with a certain accuracy and the value of the angle of elevation, while theoretical calculations, was set with an accuracy of 1 minute. The values \( t_{ik} \), \( \theta_{ic} \), \( H_i \), \( V_{ik} \), which are given in the firing tables, were determined by numerical methods and approximately [1].

At angles of elevation \( 7^\circ 57' < \alpha \leq 15^\circ 05' \) the velocity of the projectile during flight changes from supersonic to transonic velocity. Therefore, at the time when the velocity of the projectile transforms from supersonic to transonic, the projectile motion at supersonic velocity completes consequently the motion at transonic velocity begins.

The functional dependence of the magnitude of drag force at the stage of motion with transonic velocity is also described by formula (3), but with the parameters \( \gamma_2 \) and \( \beta_2 \). Their values are determined similarly to the previous one, considering the results of the training area’s studies [10]:

\[
x(0_{40}) = 7800, \quad x(0_{41}) = 8000, \ldots, \quad x(0_{56}) = 11000. \quad (21)
\]

The initial conditions for the system of differential equations (14)–(16), at the stage of flight with transonic velocity, are the values of the projectile’s kinematic parameters at the moment when its velocity is equal to the velocity of sound in the air. It combines the stages of projectile flight with supersonic and transonic velocities. The value of the coefficient, due to the aerodynamic shape of the projectile and its proportionality, was left unchanged \( c_x = 0.35 \). At the stage of flight of the projectile at transonic velocity \( \gamma_2 = -0.09 \) and \( \beta_2 = 6.548 \).

Comparing the theoretical and experimental results of the horizontal coordinate of the level point of the projectile trajectory given in Table 2, it can be argued that the differences between them are insignificant. The maximum deviation of the module is 6.5 meters.

In fig. 2 \( t \sigma_1 \) denotes the moment when the projectile’s velocity transforms from supersonic to transonic; \( t2k \) – the zeroing of the projectile’s trajectory (duration of its motion).

### Table 2

The values of the projectile’s kinematic parameters, while flying at supersonic and transonic velocities, obtained theoretically and provided in the firing tables [12]

<table>
<thead>
<tr>
<th>( \alpha_i ), (( i = 1,39 )), degree</th>
<th>( t_{ik} ), s</th>
<th>( \theta_{ic} ), degree</th>
<th>( x_{ik} ), m</th>
<th>( \dot{x}_{ik} ), m/s</th>
<th>( \dot{z}_{ik} ), m/s</th>
<th>( V_{ik} ), m/s</th>
<th>( H_i ), m</th>
<th>( x_{iH} ), m</th>
<th>( S_i ), m</th>
</tr>
</thead>
<tbody>
<tr>
<td>8°17'</td>
<td>16,916 (17)</td>
<td>12°50' (13°)</td>
<td>7802,43 (7800)</td>
<td>328,88</td>
<td>-74,97</td>
<td>337,32 (348)</td>
<td>352,2 (353)</td>
<td>4336</td>
<td>7846</td>
</tr>
<tr>
<td>11°46'</td>
<td>22,902 (23)</td>
<td>19°40' (19°)</td>
<td>9606,51 (9600)</td>
<td>289,76</td>
<td>-103,55</td>
<td>307,71 (317)</td>
<td>652,5 (651)</td>
<td>5463</td>
<td>9727</td>
</tr>
<tr>
<td>14°04'</td>
<td>26,518 (26)</td>
<td>23°47' (23°)</td>
<td>10599,6 (10600)</td>
<td>276,09</td>
<td>-121,66</td>
<td>301,70 (309)</td>
<td>886,1 (881)</td>
<td>6080</td>
<td>10800</td>
</tr>
<tr>
<td>15°05'</td>
<td>28,052 (28)</td>
<td>25°30' (25°)</td>
<td>11003,9 (11000)</td>
<td>271,14</td>
<td>-129,28</td>
<td>300,39 (307)</td>
<td>997,2 (988)</td>
<td>6326</td>
<td>11247</td>
</tr>
</tbody>
</table>

Source: developed by the authors.
At angles of elevation greater than $\alpha_{55} = 15^\circ 05'$ the projectile’s velocity, while moving, varies: firstly it is supersonic, then transonic and finally subsonic. The change in projectile’s velocity from transonic to subsonic is carried out at the time when the speed of the projectile, which was calculated using the formulas for the second stage, begins to increase.

The initial conditions for the system of differential equations (14)–(16), at the stage of projectile flight at subsonic velocity, are the values of its kinematic parameters at the time when the inequality

$$V(t) \leq V(t + \Delta t),$$

where $\Delta t$ — small magnitude.

It combines the stages of projectile flight at transonic and subsonic velocities.

The magnitude of the coefficient, due to the aerodynamic shape of the projectile and its proportionality, $c_x = 0.35$ was left unchanged. The values of the parameters $\gamma_3$ and $\beta_3$, at the stage of flight of the projectile at subsonic velocity, are equal to: $\gamma_3 = -0.196$ and $\beta_3 = 2.760$.

Comparing the theoretical and experimental results of the horizontal coordinate of the level point of the projectile trajectory given in Table 3, we can state that the differences between them are insignificant. The maximum deviation of the module is 20.1 meters.

In fig. 3, $\tau_{2V}$ is the moment at which the speed of the projectile changes from transonic to subsonic; $\tau_{3k}$ — the moment of zeroing the projectile’s trajectory. (duration of its motion).

**Remark.** The use of the averaged drag force coefficient $C_D$ in formula (2) worsens the accuracy of the description of projectile dynamics. The value of the drag force coefficient is variable, since the speed of the projectile and sound in the air (it is different at different heights) during the flight of the projectile change.

The graph of the function $CD(t) = \left(\frac{V(t)}{V_s(t)}\right)^{\beta_i}$ in the case of the angle of elevation $\theta_{55} = 37^\circ 50'$ for the D-20 howitzer with a full charge is presented in Fig. 4.

**Table 3**

<table>
<thead>
<tr>
<th>$\alpha_i$ ($i = 1, 39$), degree</th>
<th>$t_{ik}$, s</th>
<th>$\theta_i$, degree</th>
<th>$x_{ik}$, m</th>
<th>$\dot{x}_{ik}$, m/s</th>
<th>$\ddot{x}_{ik}$, m/s</th>
<th>$V_{ik}$, m/s</th>
<th>$H_i$, m</th>
<th>$x_{iH}$, m</th>
<th>$S_i$, m</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_{57} = 15^\circ 36'$</td>
<td>28,8194 (28)</td>
<td>26$^\circ 20'$ (26$^\circ$)</td>
<td>11202,4 (11200)</td>
<td>268,96</td>
<td>-133,14</td>
<td>300,11 (306)</td>
<td>1055 (1040)</td>
<td>6446</td>
<td>11469</td>
</tr>
<tr>
<td>$\alpha_{72} = 25^\circ 02'$</td>
<td>41,7859 (42)</td>
<td>39$^\circ 11'$ (39$^\circ$)</td>
<td>14213,4 (14200)</td>
<td>238,42</td>
<td>-194,34</td>
<td>307,59 (310)</td>
<td>2288 (2260)</td>
<td>8090</td>
<td>15144</td>
</tr>
<tr>
<td>$\alpha_{69} = 31^\circ 44'$</td>
<td>50,2968 (50)</td>
<td>46$^\circ 16'$ (46$^\circ$)</td>
<td>15779,9 (15800)</td>
<td>219,53</td>
<td>-229,40</td>
<td>317,52 (315)</td>
<td>3319 (3290)</td>
<td>8822</td>
<td>17474</td>
</tr>
<tr>
<td>$\alpha_{69} = 45^\circ 00'$</td>
<td>66,0208 (66)</td>
<td>57$^\circ 18'$ (58$^\circ$)</td>
<td>17424,6 (17410)</td>
<td>182,56</td>
<td>-284,37</td>
<td>337,93 (319)</td>
<td>5608 (5610)</td>
<td>9412</td>
<td>21432</td>
</tr>
</tbody>
</table>

Source: developed by the authors.
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Fig. 3. Varying in projectile velocity and the drag force at the angle of elevation $\alpha_{89} = 45^\circ 00'$:
a) – varying in projectile velocity; b) – varying in projectile drag force
Source: developed by the authors based on formulas (14)–(16) for a) and formula (12) for b).

Knowledge of the functional dependence of the value of the projectile’s drag force allows to consider the effect of deterministic and non-deterministic factors on the dynamics of the projectile’s motion.

For example, the initial speed changes by 1 %, then the increase in the flight range of the projectile, in the case of the throwing angle $\theta_{85} = 37^\circ 50'$ for the D-20 howitzer with the full charge determined theoretically is equal to 176.93 meters. According to the Shooting Table, the increase is 174 meters.

If the atmospheric pressure changes by 10 mm Hg then the increase in the flight range of the projectile determined using formula (3), is equal to 108.61 meters, and according to the Shooting Table – 105 meters.

If the air temperature changes by $+10$ °C, in the case of the angle of elevation $\theta_{44} = 9^\circ 43'$ for the D-20 howitzer, the charge is full, then the increase in the flight range of the projectile is equal to 95.85 meters, and according to the Firing Table – 91 meters.

However, there is a significant discrepancy in the increase in flight distance in the case of large angles of elevation. If this angle is equal to $\theta_{85} = 37^\circ 50'$, then the increase in the flight range of the projectile, determined using formula (3), is 356.79 meters, and according to the Shooting Table, the increase is 280 meters.

Example of a problem. Determine the angle of elevation if the target’s coordinates are $x_c = 16450$ m and $z_c = 150$ m. Assume that the mass of the projectile is 1 % greater than the standard one, the initial velocity of the projectile is less than 1 %, air temperature 35 °C, atmospheric pressure 710 mm Hg.

Due to the proposed technique and the appropriate software we obtain that in this case the angle of elevation should be equal to $\alpha = 31^\circ 17' 45''$.

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Fig. 4. Changing of the function $CD(t)$ at the angle of elevation $\theta_{85} = 37^\circ 50'$ for the D-20 howitzer with a full charge
Source: developed by the authors.

In this figure:
(0; $t_1$) is the duration of the projectile movement with supersonic speed;
($t_1; t_1 + t_V 2$) – the movement of the projectile with transonic speed;
($t_1 + t_V 2 < t$) – the movement of the projectile with subsonic speed.

Analyzing the graph, it can be argued that the value of the function $CD(t)$ varies within (0.3; 1).

Conclusions

The varieties between the theoretically determined coordinates of the level point of the projectile trajectory $x(\theta_j)$, based on the proposed mathematical model, and those given in the firing tables are insignificant, which confirms the adequacy of the proposed model.
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МАТЕМАТИЧНА МОДЕЛЬ ЗОВНІШНЬОЇ БАЛІСТИКИ СНАРЯДІВ

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У роботі запропоновано математичну модель дослідження зовнішньої балістики куль та снарядів. Ця модель ілюструється на досліджені динаміки снаряду ОФ-540 в повітрі, випущеного з гаубиці Д-20. Вона базується на використанні результатів полігонних досліджень, які наведені в таблицях стрільб. Вирішальним вплив на динаміку снаряду відіграє сила зовнішнього опору повітря. Безпосереднє визначення функціональної залежності цієї сили від сукупності детермінованих (форми і маси снаряду, густини і температури повітря, атмосферного тиску, дульної швидкості) і недетермінованих (дульної швидкості, величини і напрямку швидкості вітру) параметрів є досить проблематичним і на сьогодні не реалізоване. Автори визначають функціональну залежність, розв'язуючи обернену задачу механіки. Знайоми далість лету снаряду при певних кутах прицілювання, наведених в таблицях стрільб, визначають значення сили зовнішнього опору повітря, враховуючи дію на снаряд його ваги та Корілісової сили. У роботі враховано, що характер функціональної залежності сили зовнішнього опору повітря від швидкості є різним на етапах руху снаряду з надзвуковою, підзвуковою та дозвуковою швидкостями, і вказано умови переходу від одного етапу руху снаряду до іншого. На основі функціональної залежності сили зовнішнього опору повітря снаряду, представлених у роботі, можна визначити вплив температури заряду снаряду на його кінематичні параметри руху. Здійснено порівняння кінематичних параметрів руху снаряду, визначених методом, запропонованим авторами, з результатами, наведенями в таблицях стрільб, та вказано на основі їх розбіжності. Розбіжності між теоретичними результатами і реальними даними лету снаряду зазначаються у таблицях стрільб. Знання функціональної залежності сили зовнішнього опору повітря дозволяє автоматизувати, використовуючи відповідне програмне забезпечення, визначати кута прицілювання в залежності від місця розташування кути та значень детермінованих і недетермінованих чинників.

Ключові слова: гармата, динаміка руху снаряду, зовнішня балістика, сила зовнішнього опору повітря.